Course Outline

School Name:	KEEWAYTINOOK INTERNET HIGH SCHOOL
Department Name:	Mathematics

Ministry of Education Course Title: Functions

Grade Level: 11

Ministry Course Code: MCR3U

Teacher's Name: Sangchul Yu

Developed by: Lorne Goring   Date: February 2010

Revision Date: September 2020

Developed from:
Ontario Ministry of Education. (2007). *The Ontario curriculum, grades 11 and 12: Mathematics.* Toronto ON: Queen's Printer for Ontario.

Text: None

Prerequisite: MPM2D

Credits: One

Length: 110 hours

Principal's Name: Angela Batsford-Mermans

Principal's Approval: *Angela Batsford-Mermans*

Approval Date: September 13, 2021

Course Description/Rationale

This course introduces the mathematical concept of the function by extending students' experiences with linear and quadratic relations. Students will investigate properties of discrete and continuous functions, including trigonometric and exponential functions; represent functions numerically, algebraically, and graphically; solve problems involving applications of functions; investigate inverse functions; and develop facility in determining equivalent algebraic expressions. Students will reason mathematically and communicate their thinking as they solve multi-step problems.

Overall Curriculum Expectations

Characteristics of Functions
- Demonstrate an understanding of functions, their representations, and their inverses, and make connections between the algebraic and graphical representations of functions using transformations;
- Determine the zeros and the maximum or minimum of a quadratic function, and solve problems involving quadratic functions, including problems arising from real-world applications;
- Demonstrate an understanding of equivalence as it relates to simplifying polynomial, radical, and rational expressions.

Exponential Functions
- Evaluate powers with rational exponents, simplify expressions containing exponents, and describe properties of exponential functions represented in a variety of ways;
- Make connections between the numeric, graphical, and algebraic representations of exponential functions;
- Identify and represent exponential functions, and solve problems involving exponential functions, including problems arising from real-world applications.

Discrete Functions
- Demonstrate an understanding of recursive sequences, represent recursive sequences in a variety of ways, and make connections to Pascal's triangle;
- Demonstrate an understanding of the relationships involved in arithmetic and geometric sequences and series, and solve related problems;
- Make connections between sequences, series, and financial applications, and solve problems involving compound interest and ordinary annuities.

Trigonometric Functions
- Determine the values of the trigonometric ratios for angles less than 360⁰; prove simple trigonometric identities; and solve problems using the primary trigonometric ratios, the sine law, and the cosine law;
- Demonstrate an understanding of periodic relationships and sinusoidal functions, and make connections between the numeric, graphical, and algebraic representations of sinusoidal functions;
- Identify and represent sinusoidal functions, and solve problems involving sinusoidal functions, including problems arising from real-world applications.

Course Content

Unit	Length
1. Algebra	12 hours
2. Representing Functions	14 hours
3. Quadratic Functions	14 hours
4. Determining and Applying Trigonometric Ratios	15 hours
5. Graphing Trigonometric Functions	14 hours
6. Exponential Functions	15 hours

7. Sequences and Series	12 hours
8. Financial Mathematics	12 hours
Total	110 hours

Unit Descriptions

Unit 1 – Algebra
In this unit students will simplify polynomial expressions through addition, subtraction and multiplication. Rational expressions will be simplified by adding, subtracting, multiplying, and dividing, stating restrictions on variable values. Finally, students will determine through substitution and simplification if two given algebraic expressions are equivalent.

Unit 2 – Representing Functions
Through authentic models, students are introduced to the definition of a function and the notations associated with it. Students use graphing technology and paper and pencil tasks to investigate the properties of functions and their inverses, and the transformations of functions. The investigations are used to introduce and extend the use of function notation to inverses and transformations. Students explore the domain and range of functions, inverses, and transformations.

Unit 3 – Quadratic Functions
Students will solve problems that involve quadratic functions arising from real-world applications. They will determine the maximum or minimum value of a quadratic function and the number of zeroes (i.e., x-intercepts). Students will simplify radicals and radical expressions obtained by adding, subtracting, and multiplying.

Unit 4 – Determining and Applying Trigonometric Ratios
Students consolidate and extend concepts first introduced in Grade 10. Students use the primary trigonometric ratios, the sine law, and the cosine law to model and solve two and three dimensional problems involving acute, right, and oblique triangles. Students investigate the relationship between degree and radian measure, and explore the use of the unit circle and special triangles to determine selected values of the primary trigonometric ratios. Methods of proof are introduced and applied to verify trigonometric identities. Students develop the skills to manipulate and solve trigonometric equations.

Unit 5 – Graphing Trigonometric Functions
Students investigate the periodic nature and graphical properties of the primary trigonometric functions. Using technology, students explore the effects of simple transformations on their graphs and equations. Students apply these concepts to model authentic problems.

Unit 6 – Exponential Functions
Students will use prior knowledge of linear and quadratic functions to compare with exponential functions (comparing rates of change using finite differences in tables of values; identifying a constant ratio in a table of values, inspecting graphs, comparing equations). Students will graph various exponential functions and make comparisons between the graphs.

Unit 7 – Sequences and Series
Students will investigate arithmetic and geometric series and sequences, and then solve related problems. They will demonstrate an understanding of recursive sequences and represent them in a variety of ways, including connections to Pascal's triangle.

Unit 8 – Financial Mathematics
Students develop the formula for compound interest and solve problems related to compound interest and annuities. As skills are developed, students use spreadsheets to investigate the cost of borrowing when interest rates, compound periods, lending terms, etc., are varied. The activities are designed to reflect the type of decisions that students are likely to face in their future, in their home communities and more regionally. Students apply skills with linear and exponential functions.

Teaching/Learning Strategies

This course is organized into an eight-week series of lessons and activities that is presented to students in remote northern communities via the internet. The eighth week is used for course consolidation, review, and the final examination. Teacher and students communicate over the internet through timely activity feedback, emails, messages, video and audio calls. Classroom mentors assume the role of liaison between the teacher and student while also supporting a holistic approach to motivate, engage and support each individual student.

A variety of strategies will be used in the online delivery of this course. Some instructional strategies include:

- The use of rich contextual problems which engage students and provide them with opportunities to demonstrate learning, and appreciate the need for new skills;
- The prompting, supporting, and challenging of individual students; addressing individual students' needs and ensuring sound learning opportunities for every student.
- The use of technological tools and software (e.g., graphing software, dynamic geometry software, the Internet, spreadsheets, and multimedia, discussion forums, live chat) in activities, demonstrations, and investigations to facilitate the exploration and understanding of mathematical concepts;
- The use of learning/performance tasks that are designed to link several expectations and give the students occasion to demonstrate their optimal levels of achievement through the demonstration of skill acquisition, the communication of results, the ability to pose extending questions following an inquiry, and the determination of a solution to unfamiliar problems;
- The use of accommodations, remediation, and/or extension activities, where necessary, to meet the needs of exceptional students.
- Encourage students to investigate, to reason, to explore alternative solutions and to take the risks necessary to become successful problem solvers.

Learning goals will be discussed at the beginning of each assignment and success criteria will be provided to students. The success criteria are used to develop the assessment tools in this course, including rubrics and checklists.

Evaluation

The final grade will be determined as follows (Ontario Ministry of Education, 2010):
- Seventy per cent of the grade will be based on evaluation conducted throughout the course. This portion of the grade should reflect the student's most consistent level of achievement throughout the course, although special consideration should be given to more recent evidence of achievement.
- Thirty percent of the grade will be based on a final evaluation administered at or towards the end of the course. This evaluation will be based on evidence from one or a combination of the following: an examination, a performance, an essay, and/or another method of evaluation suitable to the course content. The final evaluation allows the student an opportunity to demonstrate comprehensive achievement of the overall expectations for the course (p. 41).

Ontario Ministry of Education. (2010). *Growing success: Assessment, evaluation and reporting in Ontario schools.* Toronto ON: Queen's Printer for Ontario.

Type of Assessment	Category	Details	Weighting (%)
Term Work (70%)	Knowledge/ Understanding	Explore exponential relations and perform algebraic operations	13

	Thinking	Investigate properties of inverse functions		19
	Communication	Verify, through investigation facilitated by dynamic geometry software, geometric properties and relationships involving functions		19
	Application	Solve exponential equations Apply exponential laws Find inverse functions by applying algebraic approaches		19
Final Evaluation (30%)	Culminating Activity (15%)	Graph quadratic functions by hand and using technology Perform transformations on plotted functions Solve quadratic equations of real world examples Calculate using Sine and cosine laws	K/U	3
			T	4
			C	4
			A	4
	Exam (15%)	Analyze various aspects on the graph of a quadratic function Compare investment options and make an educated decision Use the sine/cosine law to calculate distances Solve problems using graphical representations of data	K/U	3
			T	4
			C	4
			A	4
			TOTAL	**100**

Assessment/Evaluation Strategies

A variety of assessment and evaluation methods, strategies and tools are required as appropriate to the expectation being assessed. These include diagnostic, formative, and summative within the course and within each unit.

Assessment *for* learning and assessment *as* learning is obtained through a variety of means, including the following:
- Ongoing descriptive feedback
- Small-group conversations to develop their opinions and communication skills
- Self-assessment (weekly self-assessment of learning)
- Peer assessment
- Application in familiar settings
- Application in novel settings
- Mentor observations
- Open ended questioning
- Creation of displays and projects
- Graphic organizers
- Conversations with student on a regular basis to verbalize observations, ask questions, and clarify understanding.

Evidence of student achievement (assessment *of* learning) is collected from various sources, including the following:
- Ongoing observations of most consistent work, with consideration given to most recent work
- Conversations with students
- Summative unit activities
- Quizzes
- Culminating activity
- Exam.

Whenever possible, assessment tasks are designed in real world contexts so that students see the learning in Principles of Mathematics as meaningful and relevant and are motivated to apply their learning in an assessment situation.

The Ministry of Education's 2010 document, *Growing Success*, outlines the seven fundamental principles that guide best practice in the assessment and evaluation of students. KiHS teachers use practices that:

- are fair, transparent, and equitable for all students;
- support all students, including those with special education needs, those who are learning the language of instruction (English or French), and those who are First Nation, Métis, or Inuit;
- are carefully planned to relate to the curriculum expectations and learning goals and, as much as possible, to the interests, learning styles and preferences, needs, and experiences of all students;
- are communicated clearly to students and parents at the beginning of the course and at other points throughout the school year or course;
- are ongoing, varied in nature, and administered over a period of time to provide multiple opportunities for students to demonstrate the full range of their learning;
- provide ongoing descriptive feedback that is clear, specific, meaningful, and timely to support improved learning and achievement;
- develop students' self-assessment skills to enable them to assess their own learning, set specific goals, and plan next steps for their learning (p.6).

Resources

Ontario Ministry of Education. (n.d.). *Indigenous education strategy.*
http://www.edu.gov.on.ca/eng/aboriginal/

Ontario Ministry of Education. (2007). *The Ontario curriculum, grades 11 and 12: Mathematics.*
http://www.edu.gov.on.ca/eng/curriculum/secondary/math1112currb.pdf

Ontario Ministry of Education. (2010). *Growing success: Assessment, evaluation and reporting in Ontario schools.* http://www.edu.gov.on.ca/eng/policyfunding/growSuccess.pdf

Ontario Ministry of Education. (2016). *Ontario schools, kindergarten to grade 12: Policy and program requirements.* http://edu.gov.on.ca/eng/document/policy/os/index.html

Toulouse, P.R. (2016). What matters in Indigenous education: Implementing a vision committed to holism, diversity and engagement. https://peopleforeducation.ca/wp-content/uploads/2017/07/MWM-What-Matters-in-Indigenous-Education.pdf

Additional Resources

Britton, J. (2011, January 11). Legend of the chessboard. YouTube.
https://www.youtube.com/watch?v=t3d0Y-JpRRg

Function grapher and calculator. (n.d.). Math is Fun. https://www.mathsisfun.com/data/function-grapher.php

Further Maths. (2012, October 11). Maths tutorial: Trigonometry law of sines / sine rule. YouTube.
https://www.youtube.com/watch?v=bDPRWJdVzfs

Khan Academy. (2010, July 16). Introduction to function inverses: Functions and their graphs. YouTube. https://docs.google.com/document/d/1ARyP53E8UPGdg-8as5E8nPkh9TTbWPDA/edit

Khan, S. (n.d.). Trig word problem: modeling daily temperature. Khan Academy. https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:trig/x2ec2f6f830c9fb89:sinusoidal-models/v/modeling-with-shifted-trig-functions#

Manulife. (2012, April 20). Compound interest: The world's 8th wonder. YouTube. https://www.youtube.com/watch?v=LK7iadUBPWw

Math Meeting. (2013, June 11). Law of cosines. YouTube. https://www.youtube.com/watch?v=pxVAWYOe34c

Takota Asset Management. (2015, September 2). Simple vs. compound interest. YouTube. https://www.youtube.com/watch?v=gyiiqUQgEeA

Program Planning

This course is offered to Indigenous students living in isolated, northern Ontario communities. It is offered by qualified teachers in a blended classroom with a balance of academic, wellness, land-based learning, local language and culture to support the success of the whole student. This course uses the internet for instruction, demonstration and research. It utilizes a student-centered semi-virtual classroom which capitalizes on the strengths of internet program delivery to minimize the disadvantages of geographic remoteness.

Students are presented with 1320 minutes of instruction/activity via the internet over the period of one week. All lessons, assignments, questions and course material is presented in this manner, with approved print materials available as a student resource in each classroom. The student and instructor communicate via the internet, while a classroom mentor (a fully qualified teacher) assists students in completing tasks in a timely manner and provides support as required.

Indigenous and local content is used throughout the course to meet students' learning needs. Opportunities for outdoor activities and land-based learning are also incorporated and students are encouraged to use local knowledge in their products. Considerations are made to the learning preferences of the student population and lessons can be adjusted for individual students as required. Opportunities have been provided for students to apply ideas and concepts encountered in this course to their lives as an individual and as a member of a First Nations community. Teachers consult the Ontario Ministry of Education policies, guidelines and important initiatives when planning a comprehensive program in this area.